Quantised Hall effect and magnetoresistance through a quantum point contact

1989 J. Phys.: Condens. Matter 1 7499

(http://iopscience.iop.org/0953-8984/1/40/025)

J. Phys.: Condens. Matter **1** (1989) 7499–7503. Printed in the UK

LETTER TO THE EDITOR

Quantised Hall effect and magnetoresistance through a quantum point contact

B R Snell†, P H Beton†, P C Main†, A Neves†, J R Owers-Bradley†, L Eaves†, M Henini† and O H Hughes†, S P Beaumont‡ and C D W Wilkinson‡

† Department of Physics, University of Nottingham, Nottingham NG7 2RD, UK

‡ Department of Electronics and Electrical Engineering, University of Glasgow, Glasgow GL2 8QQ, Scotland, UK

Received 10 July 1989

Abstract. The four-terminal magnetoresistance and quantised Hall effect through a quantum point contact are investigated in a two-dimensional electron gas (2DEG) based on an n-type (AlGa)As/GaAs single heterostructure. Depending on the choice of current and voltage contacts we measure three different magnetoresistances in a quantising magnetic field. The results agree with a simple model based on conduction via edge states and also with a more conventional analysis based on the properties of a bulk 2DEG.

The two-terminal conductance of a quantum point contact (QPC) is known to be quantised both in zero and finite magnetic fields [1, 2], and is equal to $2e^2i/h$, assuming spin-degeneracy, where i is an integer. In zero magnetic field, the four-terminal conductance is equal to the two-terminal conductance except that the former is more reliably measured in practice since it eliminates problems with series lead resistances. However, in a magnetic field the situation is different. Recently, van Houten and co-workers [3] have demonstrated a negative magnetoresistance at small magnetic fields in the four-terminal resistance of a QPC. They compare their results to a simple expression derived for the four-terminal resistance

$$R_{4t} = (h/2e^2)(1/N_C - 1/N_B) \qquad N_C < N_B \tag{1}$$

where N_C is the number of conducting channels in the QPC and N_B is the number of occupied Landau levels in the bulk 2-dimensional electron gas (2DEG). Since their measurements were confined to low magnetic fields, equation (1) manifested itself as a negative 'magnetoresistance' from the zero field value of $R_{4t} = h/2e^2N_C$. In this Letter we present measurements at higher magnetic fields which confirm the validity of equation (1) in the fully quantised region where both N_B and N_C have discrete integer values. Also we find that equation (1) is only applicable when the geometrical arrangement of current and voltage leads corresponds to a measurement of ρ_{xx} in the bulk 2DEG. Measurements using lead configurations corresponding to ρ_{xy} yield two different values of resistance, depending on the direction of the magnetic field, both of which are different from R_{4t} defined in equation (1).

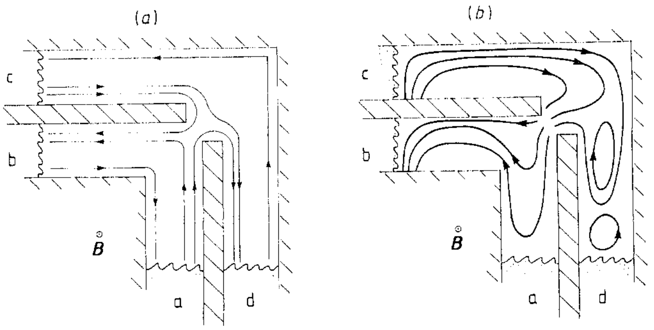

Figure 1. (*a*) Topological diagram of the sample geometry. Each contact is represented as a reservoir. The arrows on the lines representing the edge states denote the current direction for the magnetic field, ***B***, out of the paper. (*b*) The equivalent diagram for measuring *R*(cdab). Current flows between contacts c (positive) and b (earthed). The electrons flow along the voltage equipotentials in 'bulk' current-carrying states. The direction of current flow for magnetic field, ***B***, out of the paper is indicated by arrows. Near the current contacts b and c, the current flow breaks away from the equipotentials, giving rise to dissipation.

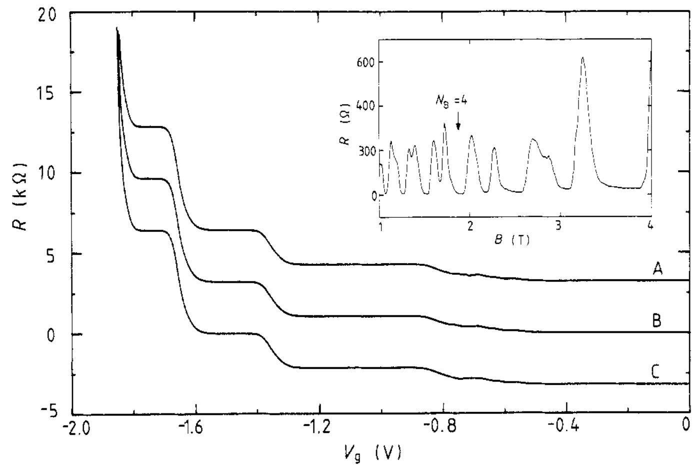

Figure 2. Four-terminal resistance measurements as a function of gate voltage, V_g, at B = 1.88 T and T = 100 mK. Curve A, *R*(cdba), curve B, *R*(cdab); curve C, *R*(dcab). Inset: *R*(cdab) as a function of magnetic field with both gates grounded.

The sample is shown schematically in figure 1. The base 2DEG material is a single heterostructure of modulation doped (AlGa)As/GaAs with sheet density $n = 3 \times 10^{11}$ cm^{-2} after illumination and a mobility of $\sim 10^6$ cm^2 V^{-1} s^{-1}, corresponding to an elastic mean free path of ~ 10 μm, as measured at 100 mK. The QPC is defined by a pair of Ti/Au gates made by electron-beam lithography and lift-off techniques. The lithographic width of each gate is 0.15 μm and they are 0.24 μm apart at their nearest points. The four ohmic contacts a, b, c, d shown in figure 1 are made using Au/Ge/Ni.

In zero magnetic field, the four-terminal conductance through the QPC shows the usual step-like variation, quantised in units of $2e/h^2$, as the voltage is varied on both gates. With both gates grounded we observe well-defined Shubnikov–de Haas oscillations, as shown in the inset of figure 2 for magnetic fields between 1 and 4 T applied perpendicular to the plane of the 2DEG. The experiments are performed at fixed values of magnetic field corresponding to the zeros in ρ_{xx}. For example, the minimum centred on 1.51 T corresponds to five spin-degenerate levels being filled. At magnetic fields above ~2 T we are able to resolve spin-splitting in the Shubnikov–de Haas oscillations, but we do not observe spin-splitting of resistance plateaux when we sweep the gate voltage.

The two-terminal magnetoresistance at all magnetic fields is given by $R_{2t} = h/2e^2N_C$ for $N_C > N_B$. The four-terminal resistance is shown in figure 2 for three different lead configurations at a magnetic field (B) of 1.88 T and a temperature of 100 mK as a function of the gate voltage. It is convenient to use the following notation to describe the current and voltage contact arrangement. Curve A is R(dcab) which is $R(I^+V^+V^-I^-)$ signifying that d and b are the two current contacts and c and a are the two voltage contacts. Note that the order of c and a is relevant. Curve B is R(cdab) and curve C is R(cdba).

Curve B corresponds to the geometry which measures ρ_{xx} in the bulk 2DEG. As can be seen in figure 2, this is the case for gate voltages, $V_g > -0.4$ V, where $R_{xx} = 0$ at 1.88 T as shown in the inset of figure 2. As the negative bias on the gates is increased and the QPC is defined, curve B follows precisely the behaviour predicted by equation 1 when $N_C < N_B$. In this case, at 1.88 T, the number of occupied spin-degenerate bulk Landau levels, $N_B = 4$. For example, the plateau centred at $V_g \sim -1.75$ V has a resistance given by equation (1) corresponding to $N_B = 4$ and $N_C = 1$. Note that the measured resistance is not the inverse of a quantised conductance value. It is the difference between two resistances, each of which is the inverse of a quantised conductance value. Curve B is also obtained when the magnetic field is reversed in direction *or* for the configuration R(dcba) which also corresponds to R_{xx} at $V_g = 0$.

Curve A follows the equation

$$\begin{aligned} R^{\alpha}_{xy} &= (h/2e^2)(1/N_C) \qquad && N_C < N_B \\ &= (h/2e^2)/(1/N_B) && N_B \leqslant N_C. \end{aligned} \tag{2}$$

Curve C, on the other hand, is described by

$$\begin{aligned} R^{\beta}_{xy} &= (h/2e^2)(1/N_C - 2/N_B) \qquad && N_C < N_B \\ &= (-h/2e^2)(1/N_B) && N_B \leqslant N_C. \end{aligned} \tag{3}$$

For example, on curve C the plateau centred at $V_g = -1.75$ V has a resistance given by equation (3) with $N_C = 1$ and $N_B = 4$. If the magnetic field is reversed then the configuration of curve A gives a resistance described by equation (3) and similarly for curve C which is then described by equation (2). Similar behaviour is observed at all magnetic fields corresponding to ρ_{xx} minima with different values of N_B but with N_C still controlled by V_g.

Following van Houten and co-workers [3] it is possible to interpret the results by assuming that in the vicinity of the QPC the current is carried by edge states [4]. Equation (1) was originally derived using this formalism [3, 5]. Each current or voltage contact is assumed to form a reservoir with chemical potential μ_i (i = a, b, c or d) as shown schematically in figure 1(a). Note that figure 1(a) is only a topological representation of the sample geometry. Using the ideas of Büttiker [5] and Landauer [6] each reservoir is assumed to inject a current of $2e\mu_i/h$ into each available conduction channel. In a magnetic field these channels may be edge states. The direction of motion of the carriers along these edge states will be determined by the direction of the applied magnetic field. In figure 1(a) the arrows show the *current* direction for $\boldsymbol{B}$ out of the paper.

Referring to figure 1(*a*), we assume there are N_B filled Landau levels in the bulk 2DEG and N_C channels through the QPC, where, for the moment, we take $N_B > N_C$. We look first at the configuration R(cdab), that is using c and b as the current leads and d and a as the voltage probes. We choose $\mu_b = 0$ for convenience. Thus to calculate the measured resistance we require

$$R(\text{cdab}) = (\mu_d - \mu_a)/eI. \tag{4}$$

The current injected by lead c is $\mu_c(2e/h)N_B$, but of this only $\mu_c(2e/h)N_C$ is transmitted to the other current lead. Thus we have

$$I = (2e/h)\mu_c N_C. \tag{5}$$

Since $\mu_b = 0$ there is no current transmitted to lead a, so that, since a is a voltage contact with zero net current, $\mu_a = 0$. It follows, therefore, that no current passes between a and d, so that the only current which enters d is the fraction of the current from c which does not go to b, i.e. $(2e/h)\mu_c(N_B - N_C)$. But d is a voltage contact with a net current of zero so

$$(2e/h)\mu_d N_B = (2e/h)\mu_c(N_B - N_C). \tag{6}$$

Combining (4), (5) and (6) we obtain

$$R(\text{cdab}) = (h/2e^2)(1/N_C - 1/N_B) \tag{7}$$

which is the result obtained by van Houten and co-workers [3]—see equation (1). Reversing the magnetic field changes the direction of the arrows on the edge states in figure 1(*a*). However, a similar analysis shows that Equation (7) is still valid, consistent with the experimental results. Likewise R(dcba) also gives the same expression. If $N_B < N_C$ then the QPC plays no part and we have a simple R_{xx} measurement and $R = 0$ for all the above cases.

We next consider R(cdba) with the magnetic field as in figure 1(*a*). Now we take $\mu_a = 0$. The current from c to b is, as above, $(2e/h)\mu_c N_C$. However, in this configuration, B is a voltage contact so the net current must be zero. Hence

$$\mu_b N_B = \mu_c N_C \tag{8}$$

and the current into a is also $(2e/h)\mu_c N_C$. No current enters d from a, since $\mu_a = 0$, so the only current entering d is the current injected by c which is not transmitted by the QPC. Hence

$$\mu_c(N_B - N_C) = \mu_d N_B. \tag{9}$$

Combining (8) and (9) we get

$$R(\text{cdba}) = (\mu_d - \mu_b)/eI = (h/2e^2)(1/N_C - 2/N_B), \tag{10}$$

as is observed in curve c of figure 2 (c.f. equation (3)).

We can also calculate R(dcab). Defining $\mu_b = 0$, it follows directly that $\mu_a = 0$. Likewise $\mu_c = \mu_d$ and

$$R(\text{dcab}) = (\mu_c - \mu_a)/eI = \mu_d/[(2e^2/h)\mu_d N_C] = (h/2e^2)(1/N_C) \tag{11}$$

in agreement with curve A of figure 2 (c.f. equation (3)). It is a simple matter to show that reversal of the magnetic field makes R(dcab) obey equation (10) and R(cdba) obey equation (11). If $N_C > N_B$ then again the QPC does not affect the measurement and

$$-R(\text{cdba}) = R(\text{dcab}) = (h/2e^2)(1/N_B) \tag{12}$$

for the direction of $\boldsymbol{B}$ implied by figure 1(*b*). Similar expressions to equations (7), (10) and (12) have been derived to describe the related experiments of Haug and co-workers [7] and Washburn and co-workers [8]. However, in their case the current carrying channels were transmitted across a 2D potential barrier rather than through a QPC.

It is interesting to note that the four-terminal resistances given by equations (7), (10) and (11) can be derived without necessarily invoking edge states by combining the two-terminal result, $R_{2t} = h/2e^2N_C$ ($N_C < N_B$), with a description [9–13] of the current flow and potential distribution in the bulk regions of the 2DEG under the dissipationless conditions of the quantum Hall effect. It is not necessary to consider the microscopic details and the current flow and potential distribution in the QPC itself. Figure 1(*b*) shows schematical 'bulk' current carrying equipotentials into and out of the QPC. The current flow crosses the equipotentials in the vicinity of the contacts b and c. The value of the two-terminal resistance between b and c gives $\mu_c - \mu_b = Ih/2e^2N_C < N_B$). In addition, $\mu_c = \mu_d$ and $\mu_a - \mu_b = Ih/2e^2N_B$. The latter result follows since a line from a and b must cross all of the current carrying equipotentials. Hence

$$\begin{aligned} R(\mathrm{cdab}) &= (\mu_d - \mu_a)/eI = (\mu_c - \mu_b)/eI - (\mu_a - \mu_b)/eI \\ &= (h/2e^2)(1/N_C - 1/N_B) \end{aligned}$$

as given by equation (7). Therefore, our four-terminal results can be deduced from the two-terminal result and a description of the quantised Hall effect without invoking edge states.

To summarise, we have measured the four-terminal resistance of a quantum point contact for various arrangements of the current and voltage contacts. We find that the results can be understood either in terms of a model involving edge states or a more conventional description of the current flow in the bulk 2DEG region.

This work is supported by SERC.

References

[1] van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouwenhoven L P, van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848
[2] Wharam D A, Thornton T J, Newbury R, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones G A C 1988 *J. Phys. C: Solid State Phys.* **21** L209
[3] van Houten H, Beenakker C W J, van Loosdrecht P H M, Thornton T J, Ahmed H, Pepper M, Foxon C T and Harris J J 1988 *Phys. Rev.* B **37** 8534
[4] Halperin B I 1982 *Phys. Rev.* B **25** 2185
[5] Büttiker M 1988 *Phys. Rev.* B **38** 9375
[6] Landauer R 1987 *Z. Phys.* B **68** 217
Landauer R 1988 *Phys. Rev. Lett.* **62** 229
[7] Haug RJ, MacDonald AH, Streda P and von Klitzing K 1989 *Phys. Rev. Lett.* **61** 2797
[8] Washburn S, Fowler AB, Schmid H and Kern D 1989 *Phys. Rev. Lett.* **61** 2801
[9] Luryi S and Kazarinov R F 1983 *Phys. Rev.* B **27** 1386
[10] Luryi S 1987 *Springer Series in Solid State Sciences* ed. G Landwehr vol 71 p 16
[11] Fang F F and Stiles P J 1984 *Phys. Rev.* B **29** 3749
[12] Zheng H Z, Tsui D C and Chang A M 1985 *Phys. Rev.* B **32** 5506
[13] Von Klitzing K and Ebert G 1984 *Springer Series in Solid State Sciences* ed. G Bauer, F Kuchar and H Heinrich vol 53 p 42